Analysis of the Influence of Phase Synchronization Error on Distributed MIMO SAR Doppler Ambiguity Eliminating

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Abstract: Phase synchronization error is one of the major problems that must be considered in distributed MIMO SAR(synthetic aperture radar, SAR) systems. Azimuth ambiguity eliminating is an important step in signal processing process, used to eliminate the azimuth spectrum ambiguity caused by low PRF. To analyze the effect caused by phase synchronization error, this paper presents the signal model under the distributed MIMO SAR system, establishes a phase synchronization error signal model, and derives the influence of phase synchronization error on signal processing. Then simulated the phase synchronization error on signal doppler ambiguity eliminating. This paper is of great significance for studying the influence of phase synchronization error on system performance under the distributed MIMO SAR system, and makes a contribution on subsequent research in error compensation methods.

1. Introduction

The spaceborne synthetic aperture radar can detect the target area all the time in space and obtain image information with high precision[1]. In order to realize people's demand for "HRWS"(high resolution wide swath, HRWS) of the required image quality, the spaceborne MIMO-SAR system has emerged. This radar transmits signals with low PRF to ensure a wide surveying band, and compensates for the lack of time sampling rate by increasing the spatial sampling rate. It utilizes multi- channel data to achieve Doppler ambiguity eliminating and obtain high-resolution images.

Distributed MIMO SAR is one type of these systems. It is characterized by placing transmitting and receiving antennas on multiple satellites with a large distance between the antennas[2]. It breaks through the limitations of the size, power consumption and volume of a single large satellite and single platform, the distributed satellite formation can effectively separate the system load, modularize the main components, and mass- produce, the development cycle is short, and the system is cost- effective[3,4].

Distributed MIMO SAR obtains more equivalent phase centers than single-transmit and single-receive after echo separation, and based on this, reconstructs the azimuth echo signal, eliminating the doppler ambiguity caused by low PRF of single satellite. But as the distance between satellites increases, the distributed transceiver platforms are relatively independent,which will bring a lot of synchronization errors. Among them, due to the different frequency sources between the transmitting and receiving radars, phase synchronization error occurs. Therefore, there are fixed frequency deviations and time- varying jitter errors between the transmitting and receiving local oscillators. The frequency deviation will directly cause the changes of signal phase, which has a great impact on the imaging quality of SAR system. Among them, the presence of phase synchronization errors will directly cause the doppler spectrum shift of the echo signal, degrades the performance of the reconstruction filter, affects the signal azimuth ambiguity performance, and worsens the imaging effect.

Based on above analysis, this article focuses on how the phase synchronization error specifically affects the azimuth doppler eliminating performance and results image quality degradation under distributed MIMO SAR system. Then this article establishes signal model with synchronization error, derives the signal formula in the process of eliminating azimuth doppler ambiguity, and simulates to testify the theory and find out imaging results under the influence of error.

2. Problem Formulation

Based on the background knowledge above, this part establishes signal model in ideal condition and real condition, which are basis for subsequent formulas derivation.

2.1. Ideal Signal Model

Taking four small satellites as an example, Figure 1 shows the working mode of distributed MIMO SAR with two transmitters and four receivers. The reference satellite R1 is located at the origin of the coordinate, the rest of the satellites are arranged along the azimuth direction, which is y-axis, the direction perpendicular to the azimuth direction is the x-axis which represents the range direction. The z-axis is the normal vector of the plane formed by x-axis and y-axis, which all constitute the $O-xyz$ plane. It is assumed that the first and the last two small satellite antennas have the function of transmitting and receiving at the same time, and the second and third small satellite antennas in the middle only have the function of receiving.

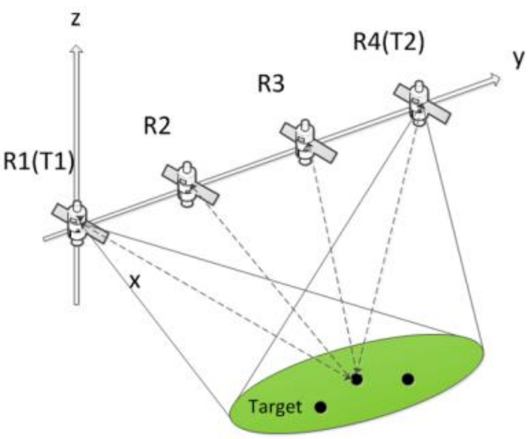


Figure 1: Two transmitters four receivers signal model.

The system works under low PRF. In a PRT time, the first and the last satellites successively transmit LFM(linear frequency modulated, LFM) signals, and each small satellite receiving channel receives two echo signals at the same time. Therefore, range ambiguity will be generated. In order to get more equivalent phase centers, echo separation should be carried out before processing signals in azimuth. Methods for separation of range echo, such as time division switching transmission, short-time shift quadrature, are proposed in reference[5,6]. After the separation of range echo, multiple echo signals with equivalent phase centers can be obtained. This paper focuses on the influence of synchronization error on azimuth signal reconstruction. The signal analysis and processing are based on the echo after range separation.

At the transmitting end, the first and last satellites transmit LFM pulses in turn, and the signal waveform of the nth pulse is:

$$S_t(\tau) = u(\frac{\tau-(n-1)T_d}{T}) \times \exp\{j2\pi f_0(\tau-(n-1)T_d) + j\pi K_r(\tau-(n-1)T_d)^2\} \quad (1)$$

Where T_d represents the pulse transmission interval. f_0 represents the center frequency, and K_r represents the frequency modulation rate of the LFM signal.

After the transmitted pulse reaches the point target, it is reflected back to the reference satellite, and the received echo signal is

$$S_r(\tau,\eta) = \iiint_{(x,y,z)} \alpha G(\eta) u(\frac{\tau-(n-1)T_d-2R_0(\eta)/c}{T}) \times \exp(j\pi K_r(\tau-(n-1)T_d-2R_0(\eta)/c)^2) \times \exp(j2\pi(-(n-1)T_d+2R_0(\eta)/\lambda)dxdydz \quad (2)$$

Where α represents the scattering intensity of the point target, which is a constant. G(η) repres ents the two-way antenna pattern of the reference satellite. Suppose that the n satellite is transmittin g and the M satellite is receiving , Rnm(η)represents the one-way equivalent slant distance of the s ignal and the η represents the slow time variable.

The Rnm(η) can be simplified as:

$$\begin{aligned} R_{nm}(\eta) &\approx \sqrt{(v\eta+(d_n+d_m)/2-y)^2+x^2+z^2} + (d_n+d_m)^2/8\sqrt{(x^2+z^2)} \\ &= \sqrt{(v(\eta+(d_n+d_m)/2v)-y)^2+x^2+z^2} + (d_n+d_m)^2/8\sqrt{(x^2+z^2)} \\ &= R_0(\eta+\eta_{nm})+\delta r_{nm} \end{aligned} \quad (3)$$

In this formula , each symbol has the following meaning:

$$R_0(\eta) = \sqrt{(v\eta-y)^2+x^2+z^2}$$

$$\eta_{nm} = (d_n+d_m)/(2v)$$

$$\delta r_{nm} = \frac{(d_n+d_m)^2}{8\sqrt{(x^2+z^2)}} \quad (4)$$

Assuming that each small satellite antenna has the same pattern, the observation signal of the m small satellite is:

$$\begin{aligned}S_{rnm}(\tau,\eta) &= \iiint_{(x,y,z)} \alpha G(\eta+\eta_{nm}) \\ &\times u\left(\frac{\tau-(n-1)T_d-2(R_0(\eta+\eta_{nm})+\delta r_{nm})/c}{T}\right) \\ &\times \exp(j\pi K_r(\tau-(n-1)T_d-2(R_0(\eta+\eta_{nm})+\delta r_{nm})/c)^2) \\ &\times \exp(j2\pi(-(n-1)T_d+2(R_0(\eta+\eta_{nm})+\delta r_{nm})/\lambda))dxdydz \\ &= S(\tau,\eta+\eta_{nm})\cdot\exp(-j\varepsilon_m)\end{aligned} \quad (5)$$

ε_m means a constant term:

$$\varepsilon_m = 4\pi\frac{\delta r_{nm}}{\lambda} = \pi\frac{(d_{T_n}+d_m)^2}{2\lambda\cdot\sqrt{(x^2+z^2)}} \quad (6)$$

2.2. Signal Model with Phase Synchronization Error

On the basis of above, this section establishes the signal model under synchronization error, and still uses the four small satellites arranged along the straight line as model.

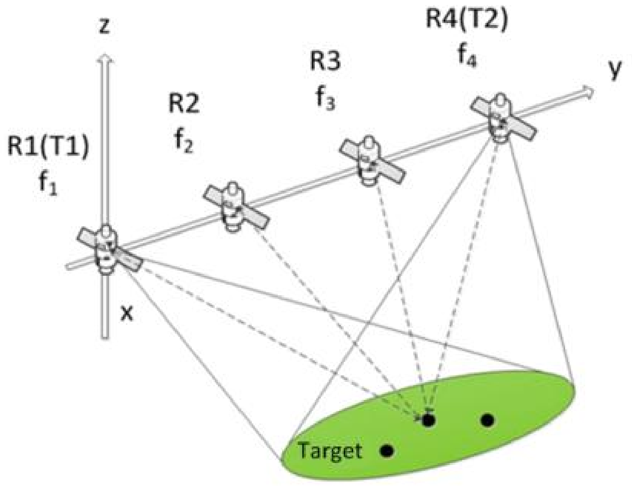


Figure 2: Signal model with different frequence sources.

However, due to the synchronization error caused by the distribution, the four satellites use different frequency sources, and the frequencies are expressed by the following formula:

$$f_i(\tau) = f_0 + \alpha_i(\tau), \quad i=1,2,3,4 \quad (7)$$

According to the expression of frequency source error, the time-varying frequency error model of frequency source can be written as the following form:

$$\alpha_i(\tau) = f_i + b_i\tau + n_i(\tau), i=1,2,3,4 \quad (8)$$

$f_i=A_if_0$ (A_i is the accuracy of the i frequency source) is the fixed frequency difference between the actual frequency value and the nominal value of the frequency source. $b_i\tau$ refers to the linear and slow drift of frequency with time caused by aging and other factors. It is described by the aging rate of the frequency source. In practical work, due to the short duration of the satellite startup, the effect on this item is negligible. The random error $n_i(\tau)$ is caused by frequency source noise varies with fast time. In conclusion, the frequency value can be expressed as,

$$f_i(\tau) = f_0 + f_i + n_i(\tau), i = 1,2,3,4 \tag{9}$$

The reference satellite emission waveform is as follows:

$$\begin{aligned} S_t^{'}(\tau) &= u(\frac{\tau-(n-1)T_d}{T}) \\ &\times \exp\{j2\pi\int_{(n-1)T_d}^{\tau} f_1(\tau)d\tau + j\pi K_r(\tau-(n-1)T_d)^2\} \end{aligned} \tag{10}$$

The reference satellite received echo signal

$$\begin{aligned} S_r^{'}(\tau,\eta) = \iiint_{(x,y,z)} & \alpha \cdot G(\eta) \cdot \\ & \times u(\frac{\tau-(n-1)T_d-2R_0(\eta)/c}{T}) \\ & \times \exp(j\pi K_r(\tau-(n-1)T_d-2R_0(\eta)/c)^2) \\ & \times \exp\{-j2\pi\int_{(n-1)T_d-2\cdot R_0(\eta)/c}^{\tau} f_1(\tau)\}dxdydz \end{aligned} \tag{11}$$

Substituting the formula (9) into (11), we get

$$\begin{aligned} S_r^{'}(\tau,\eta) = \iiint_{(x,y,z)} & \alpha G(\eta) u(\frac{\tau-(n-1)T_d-2R_0(\eta)/c}{T}) \\ & \times \exp(j\pi K_r(\tau-(n-1)T_d-2R_0(\eta)/c)^2) \\ & \times \exp(j2\pi\cdot\int_{(n-1)T_d+2\cdot R_0(\eta)/c}^{\tau}(f_0+f_i+n_i(\tau))d\tau)dxdydz \end{aligned} \tag{12}$$

Suppose that $s_0(\tau,\eta)$ represents the integral term in ideal condition. $s_1(\tau,\eta)$ and $s_2(\tau,\eta)$ represent the additional integral term caused by error.

$$\begin{aligned} S_0(\tau,\eta) &= \iiint_{(x,y,z)} s_0(\tau,\eta)dxdydz \\ &= \iiint_{(x,y,z)} \alpha G(\eta) u(\frac{\tau-(n-1)T_d-2R_0(\eta)/c}{T}) \\ & \quad \times \exp(j\pi K_r(\tau-(n-1)T_d-2R_0(\eta)/c)^2) \\ & \quad \times \exp(j2\pi\cdot(\tau-(n-1)T_d-2\cdot R_0(\eta)/c)\cdot f_0)dxdydz \end{aligned} \tag{13}$$

$$s_1(\tau,\eta) = \exp(j2\pi\cdot(\tau-(n-1)T_d-2\cdot R_0(\eta)/c)\cdot f_i) \tag{14}$$

$$s_2(\tau,\eta) = \exp(j2\pi\cdot\int_{(n-1)T_d+2\cdot R_0(\eta)/c}^{\tau} n_i(\tau)d\tau) \tag{15}$$

So the formula can be written as:

$$S_r^{'}(\tau,\eta) = \iiint_{(x,y,z)} s_0(\tau,\eta)s_1(\tau,\eta)s_2(\tau,\eta)dxdydz \tag{16}$$

Combining $s_0(\tau,\eta)$ and $s_1(\tau,\eta)$, it can be seen that the phase form of the signal has no difference from the ideal situation but only the frequency value is changed. Plus, the phase changes caused by $s_2(\tau,\eta)$ changes with time.

Then, we derive the signal form of m receiving satellite while the n satellite is transmitting with phase synchronization error.

$$\begin{aligned} S'_{rnm}(\tau,\eta) = \iiint_{(x,y,z)} & \alpha G(\eta) u\left(\frac{\tau-(n-1)T_d-2(R_0(\eta+\eta_{nm})+\delta r_{nm})/c}{T}\right) \\ & \times\exp(j\pi K_r(\tau-(n-1)T_d-2(R_0(\eta+\eta_{nm})+\delta r_{nm})/c)^2) \\ & \times\exp(j2\pi(f_0+f_i)\times(\tau-(n-1)T_d-2(R_0(\eta+\eta_{nm})+\delta r_{nm})/c)) \\ & \times\exp(j2\pi\int_{(n-1)T_d+2(R_0(\eta+\eta_{nm})+\delta r_{nm})/c}^{\tau} n_i(\tau)d\tau)]dxdydz \end{aligned} \tag{17}$$

Considering that frequency jitter error is based on zero frequency, and the fluctuation is random, so the actual integral value of the signal has little connection with time.

The formula (17) can be simplified as:

$$S'_{rnm}(\tau,\eta) = S'_r(\tau,\eta+\eta_{nm})\cdot\exp(-j\varepsilon) \tag{18}$$

In this, ε means:

$$\varepsilon = \frac{4\pi(f_0+f_i)\delta r_{nm}}{c} = \frac{4\pi\delta r_{nm}}{\lambda} \tag{19}$$

Under ideal condition,the value of frequency is f_0, now it becomes the sum of f_0 and f_i.

2.3. Azimuth Doppler Ambiguity Eliminating

In previous introduction, signal model has been established.This part mainly introduce subsequent signal processing procedure.

If the PRF (pulse repetition frequency, PRF) is f_s, the observed signal is:

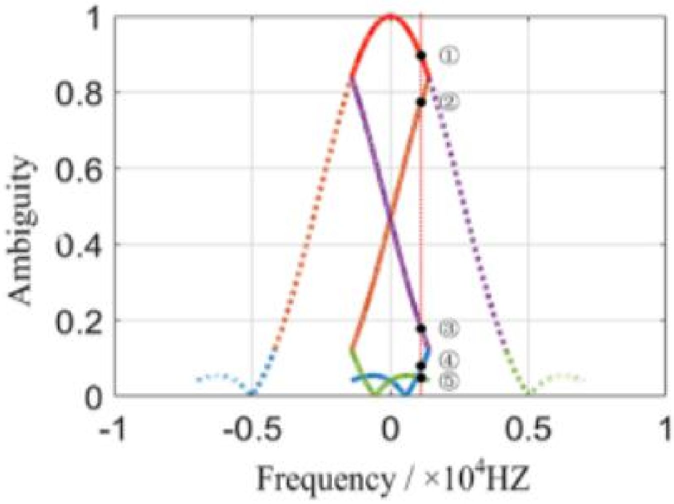
$$Z'_{rnm}(\tau,\eta) = S'_{rnm}(\tau,\eta)\sum_{i=-\infty}^{\infty}\delta(\eta-i/f_s) \tag{20}$$

The above formula is transformed by azimuth fourier transform, considering the influence of Gaussian white noise. The expression of observed signal in range doppler domain is obtained as follows:

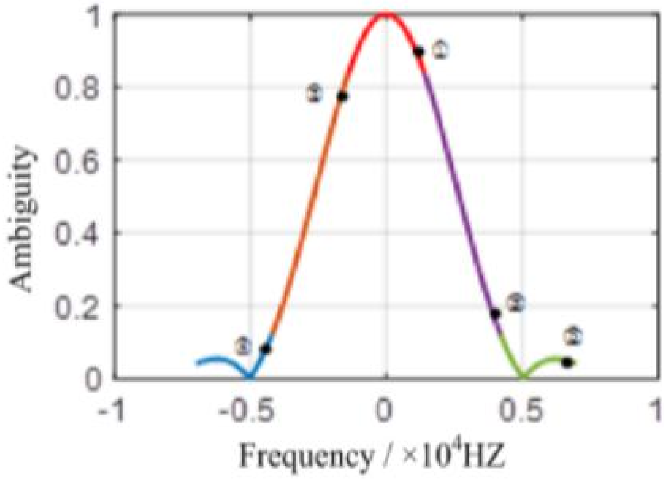
$$Z'_{rnm}(f_\eta) = \exp(-j\varepsilon)\sum_{k=-\infty}^{k=\infty} S_r(f_\eta+kf_s)\cdot\exp(j2\pi(f_\eta+kf_s)\eta_{nm}) + N_m(f_\eta) \tag{21}$$

$N_m(f_\eta)$ represents the noise spectrum of the receiving channel of m satellite.

The distributed SAR system works at low PRF, which is less than the doppler bandwidth, so there is doppler ambiguity in azimuth. Usually the system eliminates ambiguity by spatial filtering. The principle of signal reconstruction is shown in Figure 3 below. The received signal passes through the spatial filter, and the original frequency points are transformed to the corresponding azimuth frequency points, so as to achieve one- to-one correspondence between azimuth and doppler frequency, which is the unambiguous echo signal.



（a）



(b)

Figure 3: Azimuth signal reconstruction. (a) Azimuth amplitude spectrum with ambiguity, (b) Azimuth amplitude spectrum after signal reconstruction.

In this paper, we use the least square construction method. The specific principle of this method is not repeated here. Overall steps are as follows: use the signal steering vector to design a spatial filter, point to the direction of the target echo signal, suppress radar echoes in other directions, and reconstruct the selected signal in the azimuth direction to obtain an unambiguous echo signal. The steering vector and filter formula are as follows：

$$\mathbf{A}(f_\eta)=\left[\mathrm{e}^{\mathrm{j}2\pi(f_\eta+kf_s)\eta_1}\quad \mathrm{e}^{\mathrm{j}2\pi(f_\eta+kf_s)\eta_2}\quad \cdots \quad \mathrm{e}^{\mathrm{j}2\pi(f_\eta+kf_s)\eta_{2M-1}}\right]^T \tag{22}$$

$$\boldsymbol{W}(f_\eta)=\boldsymbol{A}(f_\eta)\left(\boldsymbol{A}(f_\eta)^H\boldsymbol{A}(f_\eta)\right)^{-1} \tag{23}$$

$$\boldsymbol{Z}_e(f_\eta)=\mathbf{W}^{\mathbf{H}}(f_\eta)\mathbf{Z}(f_\eta) \tag{24}$$

$\boldsymbol{Z}(f_\eta)$ is the signal matrix form observed by each equivalent channel. $\boldsymbol{Z}(f_\eta)$ represents the estimation of unambiguous signal.

With phase synchronization error, the estimation signal matrix is:

$$\mathbf{Z}_e(\tau,\eta) = \mathbf{W}^H(f_\eta)\mathbf{Z}_r(\tau,\eta) \tag{25}$$

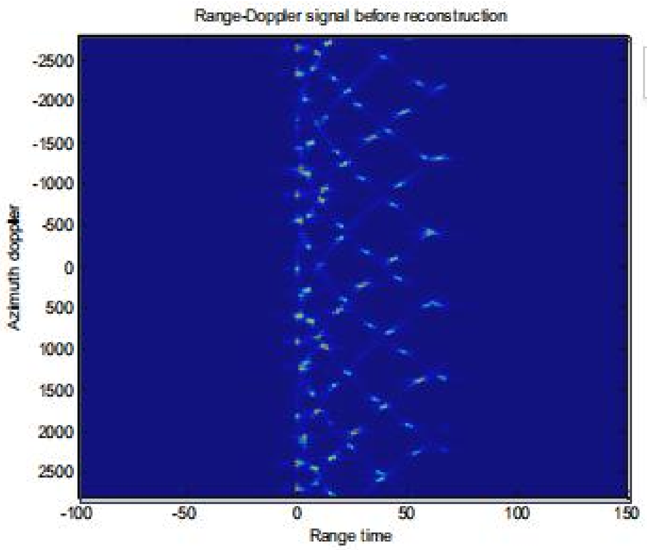
3. Experiment and Analysis

In this section, numerical simulations are conducted to find out the influence caused by phase synchronization error . The simulation parameters are listed in chart 1 below.

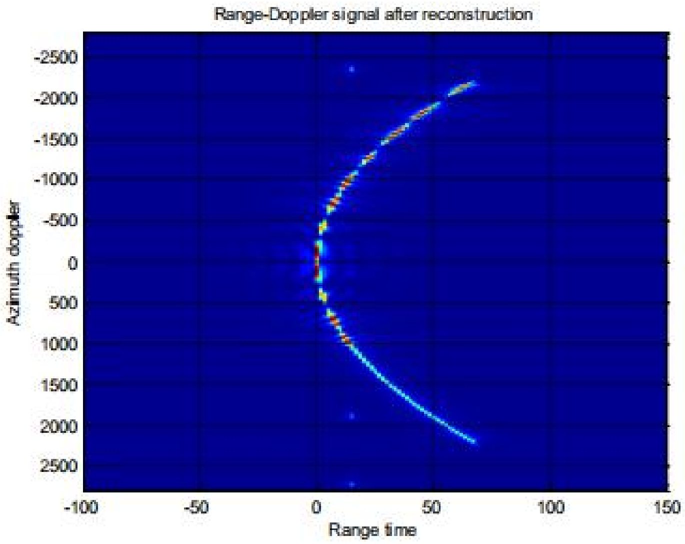
At first, simulate the signal under ideal condition, adopt the "two-transmit and four-receive" mode, and set the coordinates of two point targets to [0,0] and [0,-5000]. The distributed SAR system can obtain 7 equivalent channels and has the ability to resolve 7 times of ambiguity. When the PRF value is 998 Hz, there are 5 times azimuth ambiguity, as shown in Figure 4 a and.b. In this mode, the azimuth signal reconstruction is performed to obtain the range doppler domain signal and imaging results as shown in Figure 4 c and d. Obviously, without considering the error, the imaging quality can be obtained after azimuth doppler ambiguity eliminating.

Table 1: Distributed MIMO-SAR simulation parameters.

Parameters	Symbols	Value
Center Frequency	f_0	9.6G
Satellite Orbit Height	H	750km
Downwards Angle	Θ	35°
Squint Angle	θ_c	0°
Satellite Speed	v	7482m/s
Transmitting Antenna Size	L_t	3.0m
Distance Between Satellites	d	100m
Number of Satellites	M	4
PRF Value Range	PRF	1300~1800



(a)


Range-Doppler signal after reconstruction
Azimuth doppler
-2500
-2000
-1500
-1000
-500
0
500
1000
1500
2000
2500
-100
-50
0
50
100
150
Range time

(b)

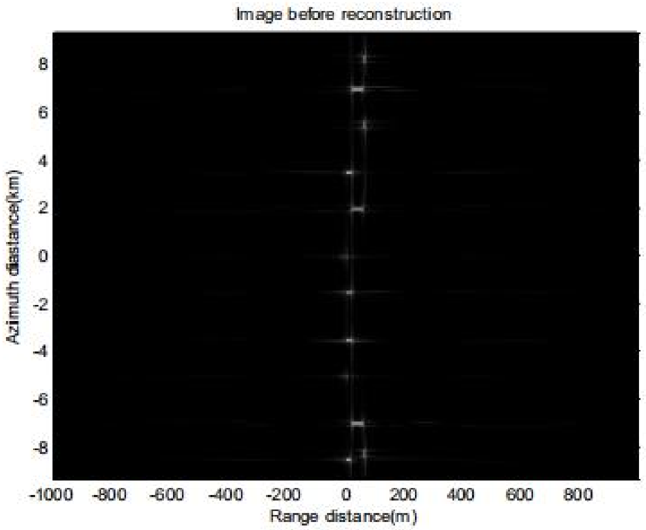


Image before reconstruction
Azimuth diastance(km)
8
6
4
2
0
-2
-4
-6
-8
-1000
-800
-600
-400
-200
0
200
400
600
800
Range distance(m)

(c)

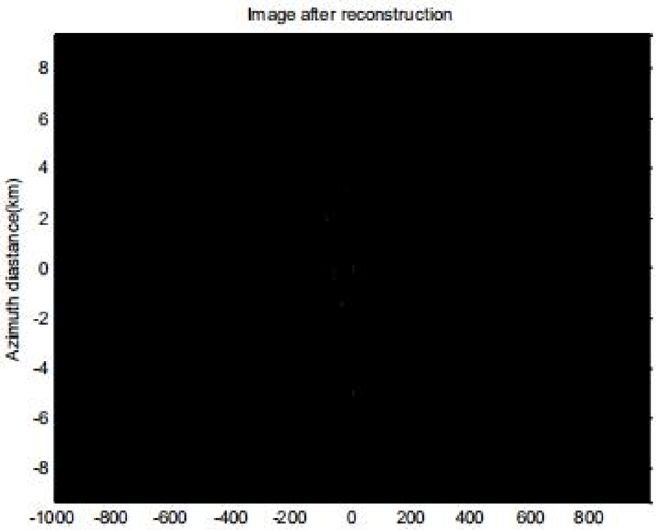
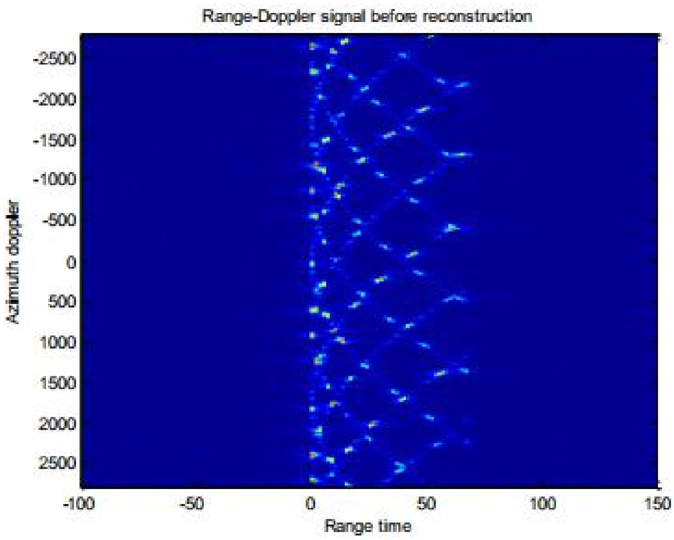


Image after reconstruction
Azimuth diastance(km)
8
6
4
2
0
-2
-4
-6
-8
-1000
-800
-600
-400
-200
0
200
400
600
800
Range distance(m)

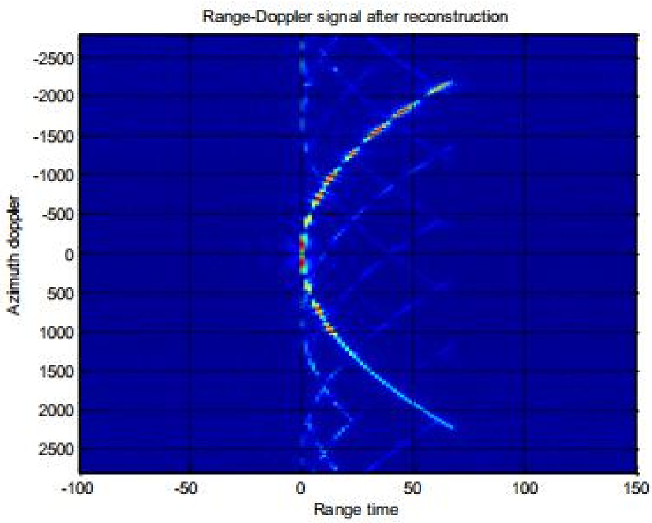
(d)

Figure 4: Imaging quality under ideal condition (a) Range-Doppler signal before reconstrction, (b) Range-Doppler signal after reconstrction, (c). Imaging effect before reconstruction, (d). Imaging effect after reconstruction.

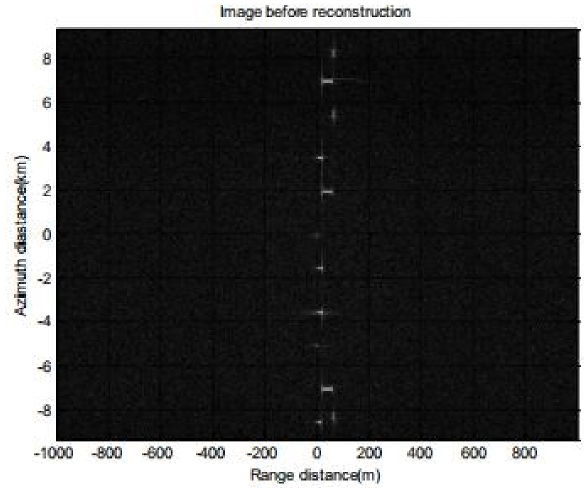
Then add the synchronization phase error to observe the reconstruction effect. First, set the fixed frequency error random frequency error as [10hz, -16hz, -20hz, 25hz] and [-1hz,1hz] respectively. At these value, the error is relatively small, but the impact on imaging quality is clearly visible. The range-doppler domain signal at the receiving end has changed due to error.



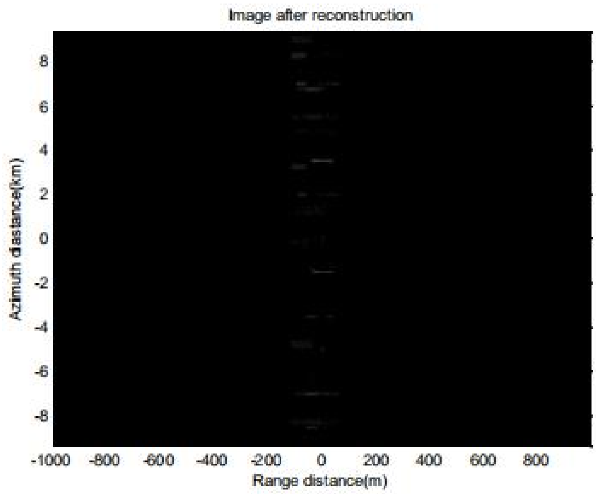
(a)



(b)



(c)

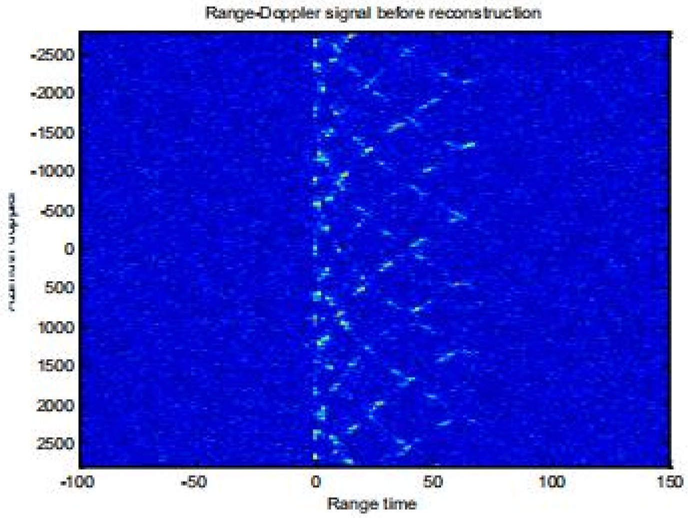


(d)

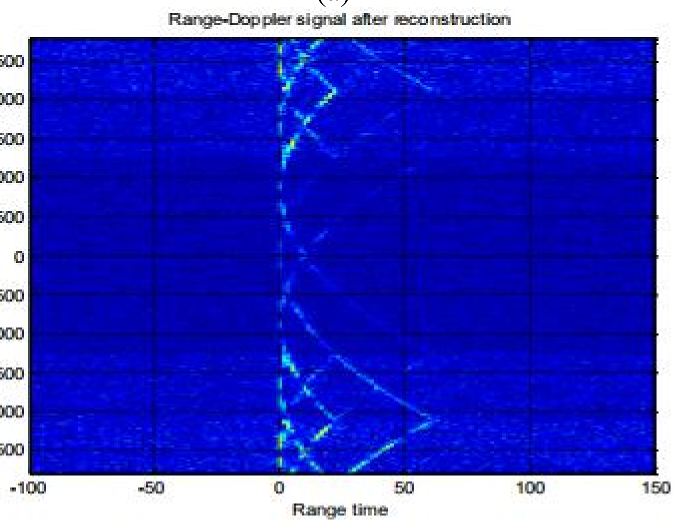
Figure 5: Imaging quality under ideal condition (a) Range-Doppler signal before reconstrction, (b) Range-Doppler signal after reconstrction, (c). Imaging effect before reconstruction, (d). Imaging effect after reconstruction.

Therefore, using an ideal signal reconstruction filter can not completely eliminate the ambiguity. It can be seen in Figure 5 c and d that although the target signal is roughly restored, there are still much ambiguity remaining and false points in the imaging result which will affect the judgment.

Then set the fixed frequency error random frequency error as [100hz,-160hz,-200hz,250hz] and [-5hz, 5hz] respectively. This time the value of error is large, which seriously affects the imaging effect. The signal reconstruction filter cannot eliminate the ambiguity, and the imaging effect is further deteriorated.


Range-Doppler signal before reconstruction
-2500
-2000
-1500
-1000
-500
0
500
1000
1500
2000
2500
-100
-50
0
50
100
150
Range time

(a)


Range-Doppler signal after reconstruction
2500
2000
1500
1000
-500
0
500
1000
1500
2000
2500
-100
-50
0
50
100
150
Range time

(b)

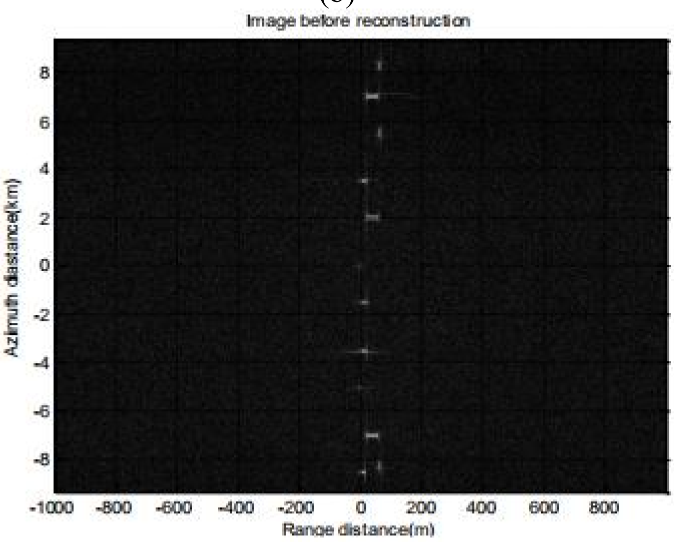
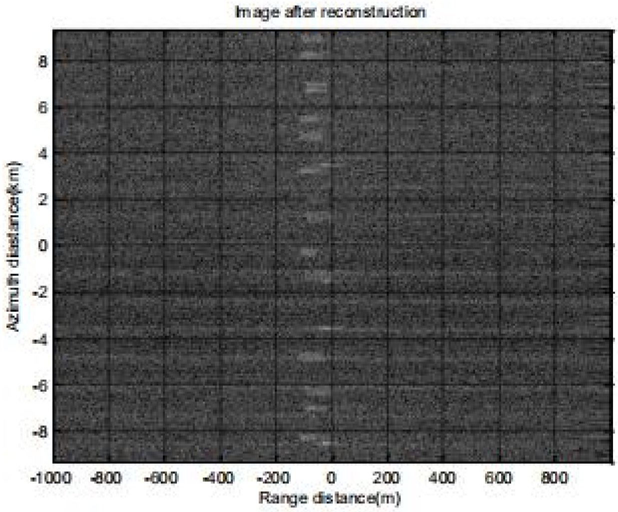


Image before reconstruction
Azimuth distance(km)
8
6
4
2
0
-2
-4
-6
-8
-1000
-800
-600
-400
-200
0
200
400
600
800
Range distance(m)

(c)



(d)

Figure 6: Imaging quality under ideal condition (a) Range-Doppler signal before reconstrction, (b) Range-Doppler signal after reconstrction, (c). Imaging effect before reconstruction, (d). Imaging effect after reconstruction.

4. Conclusions

Through formula derivation and simulation, it can be seen that the phase synchronization error directly affects the signal phase by changing the nominal frequency value. The signal changes in range-doppler domain, so the spatial filter cannot ideally reconstruct the signal. In the next step of error compensation work, the error term ε in formula 18 provides theoretical support for compensating synchronization errors under the distributed MIMO SAR system. The jitter error can be solved by time accumulation methods.

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